ERROR ESTIMATION IN DIFFERENTIAL APPROXIMATION TO EQUATION OF RADIATIVE TRANSFER

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The solution of the integrodifferential energy equation for an absorbing–emitting medium involves considerable mathematical difficulties. In view of this approximate differential equations are widely employed when computing radiative heat exchange; they contain transfer coefficients averaged over various directions [1-8]. In analyzing the range of applications of this method, the exact values of the transfer coefficients and the effect of their deviation from their mean values on the magnitude of the radiative flux must be known.

To evaluate one-dimensional radiative fluxes the following differential equation was obtained in [9] together with its boundary conditions:

$$\partial^2 q/\partial\eta^2 - [\tau_0(\tau_0 + \delta\beta l)/A]q - 4\tau_0 n^2 \partial E/\partial\eta = 0; \quad (1)$$

$$(1/\varepsilon - 1/2)q - (1/2\tau_0 m_1)\partial q/\partial\eta = (1 - 2/m_1)n^2 E \ (\eta = 0); \quad (2)$$

$$q + (1/\tau_0 m_2)\partial q/\partial\eta = 4n^2 E/m_2 - 2(1 - r)E_0 \ (\eta = 1); \quad (3)$$

$$A = \frac{\int\limits_{(4\pi)} \frac{\partial I(\eta, s)}{\partial s}\cos(s,y)d\omega_s}{\int\limits_{(4\pi)} \frac{\partial I(\eta, s)}{\partial y}d\omega_s}, \quad \delta = 1 - \frac{1}{4}\int\limits_0^\pi \gamma(\vartheta)\sin 2\vartheta d\vartheta,$$

$$m_1 = \frac{\int\limits_{(4\pi)} I(0, s)\,d\omega_s}{\int\limits_{(4\pi)} I(0, s)\,|\cos(s, y)|\,d\omega_s}, \quad m_2 = \frac{\int\limits_{(4\pi)} I(1, s)\,d\omega_s}{\int\limits_{(4\pi)} I(1, s)\,|\cos(s, y)|\,d\omega_s},$$

where q is the dimensionless radiative flux, $q = Q/\sigma T_c^4$ (Q is the resultant radiative flux; T_c is the characteristic temperature); η is the dimensionless coordinate, $\eta = y/l$; τ_0 is the optical thickness of layer of thickness l; β is the scattering coefficient; n is the refraction coefficient; E is the dimensionless hemispheric radiation density of an absolutely black body; ε is the degree of plate blackness; r is the reflection coefficient; $I(\eta, s)$ is radiation intensity at the point η in the s direction; γ is the axially symmetrical scattering indicatrix; and σ is the Stefan–Boltzmann constant.

Equations (1)-(3) are exact and take into account the anisotropy as well as the scatter of radiation. However, their solution presents difficulties, since the values of the transfer coefficients A, m_1, m_2 are not known in advance. By setting $A = 1/3$, $m_1 = m_2 = 2$, one obtains equations of differential approximation which can easily be solved. It can be shown that the above-adopted values of the coefficients correspond either to spherical isotropy of the radiation field or to isotropy of the upper or lower hemisphere. For optically dense media with moderate temperature gradients the radiation anisotropy is not high [10] and the values of the transfer coefficients remain fairly close to their mean values. A reduction in optimal thickness strengthens the radiation anisotropy and the coefficients A, m_1, m_2 start to depend on the relative intensity distribution of radiation in different directions. In this article a numerical investigation of this dependence is carried out as well as a comparison of the magnitude of radiative fluxes obtained from the exact solution by using the equations of differential approximation.

The investigation was carried out on the physical model shown in Fig. 1. A flat layer of radiative absorbing medium optically bounded by a nontransparent plate 1 and a transparent plate 2 is irradiated from above by a diffusion-radiation source of temperature Θ_0 ($\Theta = T/T_c$ is the dimensionless temperature). The

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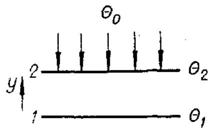

Fig. 1

TABLE 1

τ_0	τ	$\Theta_0=1,3$					$\Theta_0=1,6$				
		A	q_1	q	m_1	m_2	A	q_1	q	m_1	m_2
0,1	0		1,172	1,183				2,836	2,909		
	0,005	0.25	1,204	1,218			0,23	2,914	2,996		
	0,035	0,15	1,327	1,362	1,90	1,99	0,15	3,181	3.317	1,89	1,99
	0,065	0,16	1,377	1,427			0,18	3,258	3,435		
	0,095	0,12	1,445	1.508			0,12	3,455	3,666		
	0,100		1.466	1,529				3,524	3,740		
0,4	0		0,9593	0,9586	1,84	2,02		2,338	2,344	1,84	2,00
	0,02	0,28	1,054	1,066			0,25	2.570	2,607		
	0,14	0,21	1,269	1,338			0,21	2,987	3,151		
	0,26	0,26	1,290	1,383			0,28	3,002	3,220		
	0.38	0,17	1,412	1,515			0,17	3,332	3,588		
	0.40		1,486	1,589				3,580	3,841		
0,8	0		0,7686	0,7580	1,82	2,01		1,886	1,865	1,81	2,00
	0,04	0,25	0,9102	0,9204			0,22	2,235	2,263		
	0,28	0,25	1,141	1,220			0,26	2,665	2,837		
	0,52	0,30	1,153	1,245			0,31	2,675	2,876		
	0,76	0,23	1,301	1,399			0,25	3,044	3,279		
	0,80		1,439	1,537				3,501	3,746		
1,0	0		0.6950	0,6827				1,709	1,685	1,81	2,00
	0.05	0,24	0.8507	0,8621	1,82	2,01	0,22	2,096	2,126		
	0,35	0,27	1,078	1,158			0,28	2,515	2,688		
	0.65	0,31	1,091	1.175			0,32	2,529	2,709		
	0,95	0,25	1,245	1,334			0,29	2,904	3,117		
	1,00		1,413	1,504				3,455	3,683		
3,0	0		0,3124	0,2988	1,82	1,98		0,7776	0,7508	1,77	1,97
	0,15	0,24	0,4886	0,5037			0,23	1,223	1,264		
	1,05	0,32	0,6444	0,6723			0,33	1,509	1,567		
	1,95	0,34	0,6724	0,6719			0,34	1,563	1,550		
	2,85	0,45	0,8429	0,8648			0,72	1,915	1,944		
	3.00		1,253	1,303				3,188	3,314		
5,0	0		0,1754	0,1649	1,81	1,97		0,4398	0,4157	1,78	1,95
	0,25	0,25	0,3208	0,3362			0,25	0,8084	0,8495		
	1,75	0,33	0,4276	0,4362			0,33	1,010	1,030		
	3,25	0,34	0,4671	0,4563			0,34	1,110	1,089		
	4,75	0,57	0,6443	0,6473			1,07	1,419	1,381		
	5,00		1,218	1,287				3,142	3,328		

plates are maintained at constant temperatures equal to Θ_1 and Θ_2, respectively. The heat transfer within the layer takes place by radiation and by molecular heat conduction. The calculations are carried out for various optical thicknesses and temperatures of the outer source, the other parameters having the following values: $\Theta_1 = 0.7$; $\Theta_2 = 1.0$; $\varepsilon = 0.5$; $n = 1$; $\beta = 0$; $r = 0$; $I_W = \sigma T_C^3 l/\lambda = 10$ (I_W is the Ivanov criterion; λ is the coefficient of thermal conductivity). The temperature distribution in the layer under stationary heat conditions was evaluated on an electronic computer by using a program kindly put at our disposal by the authors of [11]. The following relations were used for the coefficients A, m_1, m_2:

$$A = \frac{-B_1 K_3(\tau) + B_2 K_3(\tau_0 - \tau) + \int\limits_0^{\tau_0} F(t) K_2 |\tau - t|\, dt}{B_1 K_1(\tau) + B_2 K_1(\tau_0 - \tau) + \int\limits_0^{\tau_0} F(t) K_0 |\tau - t|\, dt}, \tag{4}$$

$$F(t) = \begin{cases} -\Theta^4(t), & t \leqslant \tau, \\ \Theta^4(t), & t > \tau; \end{cases}$$

$$m_1 = \frac{2\left(B_1 + B_2 K_2(\tau_0) + \int\limits_0^{\tau_0} \Theta^4(t) K_1(t)\, dt\right)}{B_1 + 2B_2 K_3(\tau_0) + 2\int\limits_0^{\tau_0} \Theta^4(t) K_2(t)\, dt}; \tag{5}$$

$$m_2 = \frac{2\left(B_2 + B_1 K_2(\tau_0) + \int\limits_0^{\tau_0} \Theta^4(t) K_1(\tau_0 - t)\, dt\right)}{B_2 + 2B_1 K_3(\tau_0) + 2\int\limits_0^{\tau_0} \Theta^4(t) K_2(\tau_0 - t)\, dt}, \tag{6}$$

where

$$K_i(t) = \int\limits_0^1 \mu^{i-2} e^{-t/\mu} d\mu,\ B_2 = \Theta_0^4,$$

$$B_1 = \varepsilon\Theta_1^4 + 2(1-\varepsilon)\left(B_2 K_3(\tau_0) + \int\limits_0^{\tau_0} \Theta^4(t) K_2(t)\, dt\right).$$

The integrals appearing in (4)-(6), namely,

$$\int\limits_0^{\tau_0} F(t) K_0 |\tau - t|\, dt,\ \int\limits_0^{\tau_0} \Theta^4(t) K_1(t)\, dt,\ \int\limits_0^{\tau_0} \Theta^4(t) K_1(\tau_0 - t)\, dt,$$

are improper integrals of the second kind and depend on a parameter. Their convergence can be proved by using the Cauchy test.

To calculate the exact values of the dimensionless radiative fluxes the following relation was used:

$$q_1 = 2\left(B_1 K_3(\tau) - B_2 K_3(\tau_0 - \tau) - \int\limits_0^{\tau_0} F(t) K_2 |\tau - t|\, dt\right). \tag{7}$$

All the integrals appearing in the expressions (4)-(7) were evaluated by using the Gauss quadrature formula. Equations (1)-(3) (for $A = 1/3$, $m_1 = m_2 = 2$) were solved by using finite-differences. Some results are shown in Table 1 (the absolute values of the radiative fluxes are shown).

Analyzing the obtained results, one finds that a modification of the layer optical thickness has only a slight effect on the values of the coefficients m_1, m_2, and a stronger effect on the coefficient A. In the region of low optical thickness the values of the coefficient A may vary by almost 300% from the averaged value; however, the difference between the exact and the approximate solution is slight (never exceeding 9%). One can explain this as follows. For $\beta = 0$ the second term in Eq. (1) is of the order $0(\tau_0^2)$, and the third, $0(\tau_0)$. For $\tau_0 \ll 1$ the second term is negligible, and an inexact value of the coefficient A does not result in a great error in the magnitude of the radiative flux. The applicability of the differential approximation in the region of low optical thickness was also noted in [7, 12, 13]. With the layer optical thickness increasing the value of A comes close to 1/3. For $\tau_0 > 1$ the exact and approximate solutions are practically identical; the error increases somewhat (up to 7%) only near the boundaries where the radiation anisotropy is more essential. In the investigated range of optical thickness the temperature change of the outer sources has no noticeable effect on the error. Calculation results show that the increase in the values of the coefficients A and m_2 in Eqs. (1)-(3) results in higher absolute values of the radiative flux, and the increase in the coefficient m_1 lowers the value of $|q|$.

The obtained results demonstrate that the differential approximation remains valid for a wide range of optical thicknesses and can be applied successfully in the calculations of radiative or complex heat exchange.

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CHANGE IN AMPLIFICATION FACTOR IN THE SHOCK LAYER WHEN A SUPERSONIC FLOW WITH AN INVERTED POPULATION FLOWS AROUND BLUNT BODIES

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INTRODUCTION

In modeling the supersonic flow of a relaxing gas around a solid body it is important to make a detailed physicochemical analysis of the internal structure of the flow. The working gas used to simulate the real flow often comprises gas mixtures obtained by the combustion of hydrocarbon fuels and contains CO_2, N_2, O_2, and H_2O molecules. Of special interest in simulation problems is the circumfluence of a nonequilibrium flow with an inverted population of the vibrational levels of the CO_2 molecules. A calculation of the amplification factor for the (00⁰1) – (10⁰0) transition of the CO_2 molecule during the development of indirect jumps in compression (shock waves) in an inverted medium was presented in [1]; there was a reduction in amplification factor for the vibrational – rotational transition P(20) over the pressure range in which the greatest contribution to spectral-line broadening was due to the collision mechanism. A fall or rise in amplification factor was observed in [2], according to the intensity of the shock wave and the rotational quantum number.

In this paper we shall study the changes taking place in the amplification factor when blunt solids are immersed in a gas flow (both in the subsonic and in the supersonic parts of the shock wave) as the angle of inclination of the shock wave to the direction of the incident flow varies from 90° to the Mach angle; we shall also study the influence of small perturbations traveling through the inverted medium on the amplification factor.

§1. It is well known [3] that for the vibrational – rotational transition (00⁰1) – (10⁰0) the amplification factor of a weak signal may be written

$$G = (\lambda^2 A_{nm}/8\pi\sqrt{\pi}c)[N_n - (g_n/g_m)N_m](a/\Delta_c)H(a,0), \tag{1.1}$$

where λ is the wavelength of the transition; A_{nm} is the Einstein coefficient for the spontaneous transition $n \to m$; c is the velocity of light; the parameter $a = (\Delta_c/\Delta_D)\sqrt{\ln 2}$; Δc is the half-width of the line accounted for by collisions; Δ_D is the Doppler half-width; N_n, N_m, g_n, g_m are the populations and statistical weights of the upper and lower levels, respectively; and $H(a, 0)$ is the Voigt function in the center of the line. The temperature dependence of the shock half-width was taken as proportional to $T^{-1/2}$.

Let us consider the axisymmetrical passage (around a cylinder with spherically blunted ends) of a nonviscous supersonic homogeneous flow of relaxing gas mixture with an inverted population in the incident flow

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